Statistical Sampling Plan for the
1999 Lower Granite Dam Hydroacoustic Studies

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8 April 1999

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I. Introduction

During 1999, fixed-location hydroacoustic investigations will be performed during the spring outmigration (15 April - 31 May). This report summarizes the key statistical analyses planned for the single-beam hydroacoustic research at Lower Granite Dam in 1999. This report covers the analyses for the following three work elements:

1. Estimating spill effectiveness (SE) and spill efficiency (SF) across the project.
2. Estimating efficiency and effectiveness of the surface bypass collector (SBC).
3. Spillbay efficiency and effectiveness.

These statistical plans will be reviewed by the staff of the US Army Corps of Engineers (USACE), Battelle Pacific Northwest Division, Gary Johnson, and independent statistical reviewers employed by the USACE.

II. Transducer Deployment

A. Spillway

Each of the spillbays 2-8 will receive one single-beam transducer (Figure 1). Each transducer will be centered in the spillbays. For spillbays 2 and 6, a 12° downlooking transducer will be deployed. At the remaining spillbays 3, 4, 5, 7, and 8, a 15° downlooking transducer will be deployed. At each location, three 2.5-minute time intervals will be sampled and processed per hour in a systematic sampling scheme.

B. Turbines

Sampling at the six Lower Granite turbine units will vary over the course of the season. There will be three distinct survey periods as follows:

a. Pre-BGS trials: 15 April – 1 May
b. BGS trials: 2 – 11 May
c. Post-BGS trials: 12 – 31 May

During pre- and post-BGS trial periods, turbine passage will be monitored using one pier-nose transducer per unit. The six pier-nose transducers will be randomly positioned as much as possible to

Figure 1. Schematic showing placement of the single-beam transducer in a spillbay at Lower Granite Dam in 1999.

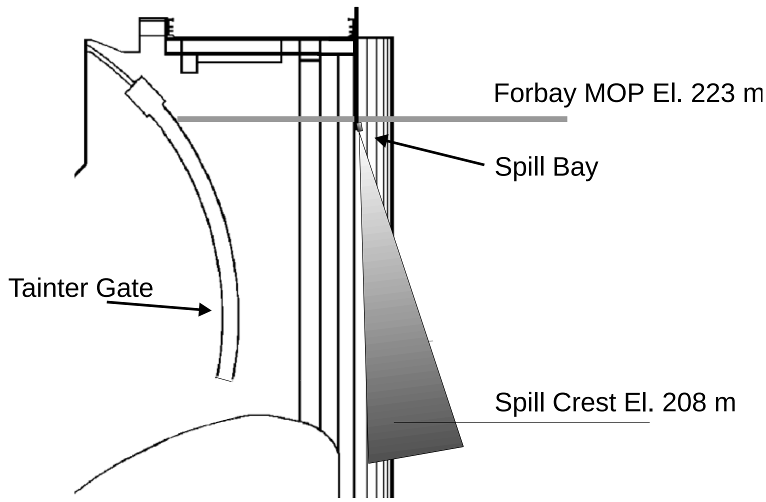
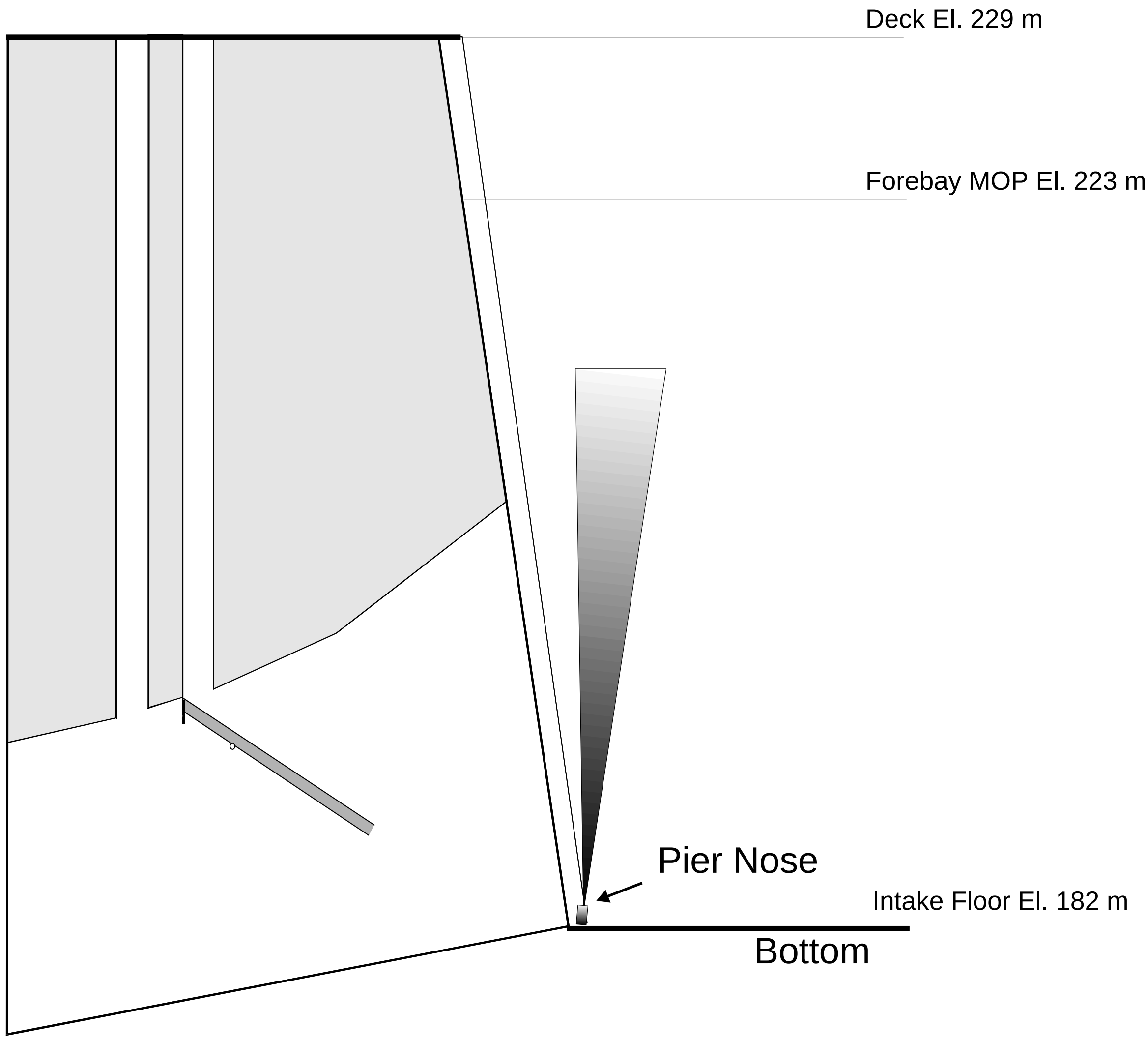


Figure 2. Schematic of pier-nose transducers placement for estimating turbine passage in 1999.



cover either A or B or C strata (Figure 2). During the BGS trial period, turbine units 1-3 will be sampled with transducers in each of the A, B, and C slots. The remaining units 4-6 will be sampled with one pier-nose transducer each as planned for the pre- and post-BGS trial periods. Each of the unit-hours will be surveyed using systematic sampling (SYS) with three 2-minute time intervals sampled and processed.

C. Surface Bypass Collector

At the surface bypass collector (SBC) in 1999, modules 3-4 and 13-14 will have openings for smolt passage. For each opening, 2 squinted-pairs of uplooking single-beam transducers will be positioned for monitoring smolt passage into and out of the SBC (Figure 3). Hence, a total of 4 squinted-pairs of transducers will be used in monitoring SBC smolt passage. Each squinted-pair of transducers will be sampled and processed three 2.5-minute time intervals per hour.

III. Estimating Smolt Passage

A. Estimating Smolt Passage at Spillway

Treating each spillbay-hour as a separate stratum and assuming simple random sampling (SRS) within the hour, the estimate of total spillway passage can be estimated as follows:

$$\hat{S} = \sum_{i=1}^{7} \sum_{j=1}^{D} \sum_{k=1}^{24} \frac{N_S}{n_S} \sum_{l=1}^{n_S} w_{ijkl}$$

where

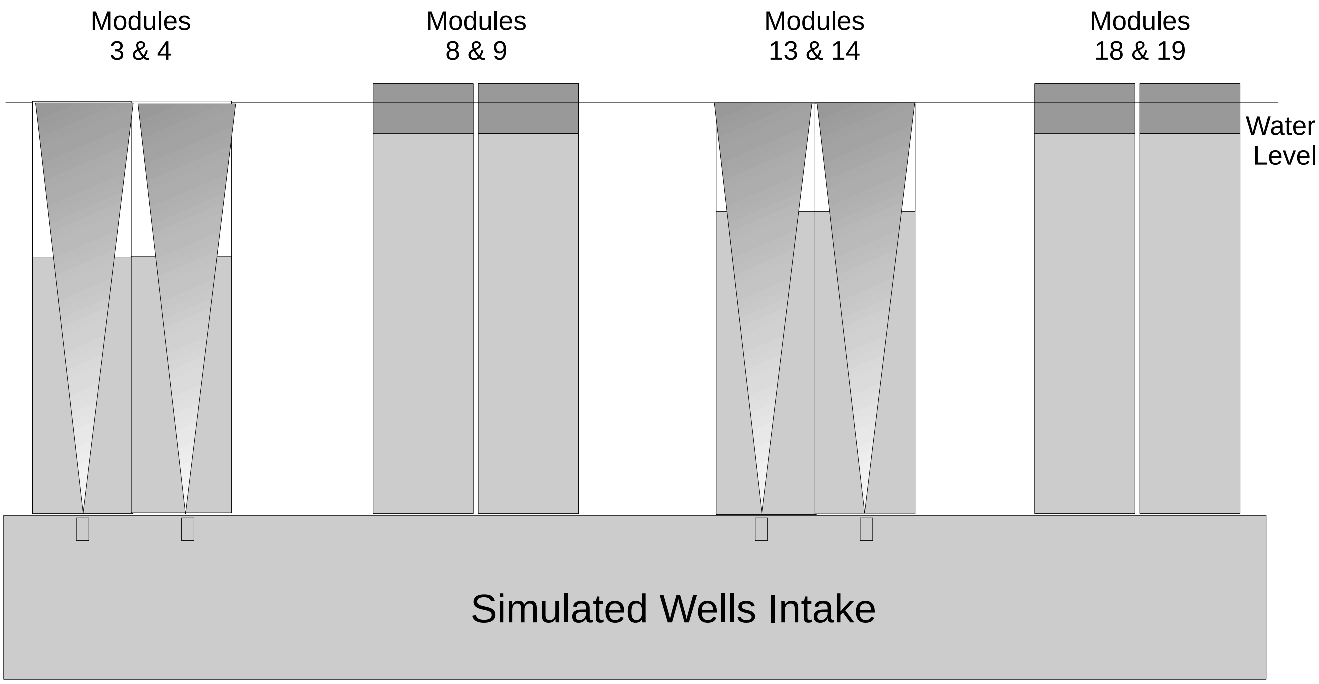
w_{ijkl} = estimated number of weighted fish in the *l*th sampling interval $(l = 1, \ldots, n)$ of the *k*th hour $(k = 1, \ldots, 24)$ of the *j*th day $(j = 1, \ldots, D)$ at the *i*th spillbay $(i = 1, \ldots, 7)$;

n_S = number of sampling intervals insonified during an hour at a spillbay;

N_s = number of possible sampling intervals within an hour at a spillbay.

Nominally, n_S = 3 2.5-minute periods out of a possible N_S = 24 time intervals per hour.

Figure 3. Schematic of transducer placement in the SBC during 1999.



The variance of $\left(\hat{S}\right)$ can be approximated by

$$V\hat{a}r\left(\hat{S}\middle|S\right) \doteq \sum_{i=1}^{7}\sum_{j=1}^{D}\sum_{k=1}^{24}\left[\frac{N_S^2\left(1-\dfrac{n_S}{N_S}\right)s_{w_{ijk}}^2}{n_S}\right]$$

where

$$s_{w_{ijk}}^2 = \frac{\sum_{l=1}^{n_s}\left(w_{ijkl}-\overline{w}_{ijk}\right)^2}{\left(n_s-1\right)},$$

$$\overline{w}_{ijk} = \sum_{l=1}^{n_s}\frac{w_{ijkl}}{n_s}.$$

B. Estimating Smolt Passage at Powerhouse

1. *Pre- and Post-BGS Trial Periods*

Treating each turbine unit-hour as a separate stratum and assuming SRS within the hour, the estimate of total powerhouse (i.e., turbine) passage can be estimated as follows:

$$\hat{T} = \sum_{h=1}^{6}\sum_{i=1}^{D}\sum_{j=1}^{24}\frac{K}{k}\sum_{l=1}^{k}x_{hijl} \tag{1}$$

where

x_{hijl} = weighted number of fish detected in the lth sampling interval $(l = 1,\ldots,k)$ in the jth hour $(j = 1,\ldots,24)$ of the ith day $(i = 1,\ldots,D)$ for the hth turbine unit$(h = 1,\ldots,6)$;

K = number of possible sampling intervals within an hour at a turbine unit;

k = number of sampling intervals insonified during an hour at a turbine unit.

Nominally, k = 3 2-minute sampling intervals out of a possible K = 30 per hour.

The estimated variance for $\hat{T}$ will be based on stratified simple random sampling formula of the form

$$\hat{Var}\left(\hat{T}\middle|T\right)=\sum_{h=1}^{6}\sum_{i=1}^{D}\sum_{j=1}^{24}\frac{K^2\left(1-\frac{k}{K}\right)s_{hij}^2}{k} \tag{2}$$

and where

s_{hij}^2 = variance among weighted fish counts within the *j*th hour $(j=1,\ldots,24)$ of the *i*th day $(i=1,\ldots,D)$ for the *h*th turbine unit $(h=1,\ldots,6)$;

$$=\frac{\sum_{l=1}^{k}\left(x_{hijl}-\bar{x}_{hij}\right)^2}{(k-1)};$$

and

$$\bar{x}=\frac{\sum_{l=1}^{k}x_{hijl}}{k}.$$

The x_{hijl} are the weighted fish counts after expanding to the area of the turbine intake. Hence, the estimate accounts for the subsampling of the turbine intake as well as subsampling over time. However, the variance cannot explicitly account for slot-to-slot variance without transducers in multiple intakes per turbine unit. As such, variance estimator (2) will underestimate the true sampling variance.

An alternative mean of estimating of the variance of $\hat{T}$ is to consider adjacent turbine units (i.e., 1-2, 3-4, 5-6) as forming spatial strata. The six slots within the two adjacent turbine units would represent the first stage of sampling. Two of the six adjacent slots will be assumed to be randomly sampled in the first stage of sampling. The second stage is the subsampling of time within a slot location. In this case of a conceptual two-stage sampling design, define

x_{ghijl} = weighted number of fish detected in the *l*th sampling interval $(l=1,\ldots,k)$ in the *j*th hour $(j=1,\ldots,24)$ of the *i*th day $(i=1,\ldots,D)$ for the *h*th turbine slot $(h=1,\ldots,6)$ of the *g*th stratum $(g=1,\ldots,3)$.

The estimator of total powerhouse passage $\left(\hat{T}\right)$ does not change (Equation 1). However, the variance of $\hat{T}$ would now be estimated by the expression

$$V\tilde{a}r\left(\hat{T}\middle|T\right)=\sum_{g=1}^{3}\frac{6^2\left(1-\frac{2}{6}\right)s_{\hat{T}_{gh}}^2}{2}+\sum_{g=1}^{3}6\left[\frac{\sum_{h=1}^{2}V\hat{a}r\left(\hat{T}_{gh}\middle|T_{gh}\right)}{2}\right] \tag{3}$$

where

$$s_{\hat{T}_{gh}}^2=\frac{\sum_{h=1}^{2}\left(\hat{T}_{gh}-\bar{\hat{T}}_g\right)^2}{(2-1)},$$

$$\bar{\hat{T}}_g=\frac{\sum_{h=1}^{2}\hat{T}_{gh}}{2},$$

$$V\hat{a}r\left(\hat{T}_{gh}\middle|T_{gh}\right)=\sum_{j=1}^{D}\sum_{k=1}^{24}\left[\frac{K^2\left(1-\frac{k}{K}\right)s_{x_{ghij}}^2}{k}\right],$$

and where

$$s_{x_{ghij}}^2=\frac{\sum_{l=1}^{h}\left(x_{ghijl}-\bar{x}_{ghij}\right)^2}{(k-1)},$$

$$\bar{x}_{ghij}=\frac{\sum_{l=1}^{h}x_{ghijl}}{k}.$$

Variance estimator (3) will tend to overestimate the true sampling variance of $\hat{T}$. Use of variances (2-3) will provide one opportunity to bracket the actual precision of the powerhouse sampling.

2. ***BGS Trial Period***

During the BGS trials, turbine units 1-3 will be sampled more intensely than turbine units 4-6. The estimation scheme should therefore take advantage of the addition precision obtained by sampling 3 of 3 turbine slots in units 1-3. For units 1-3, define

x_{cdijl} = weighted number of fish detected in the lth sampling interval $(l = 1,\ldots,k)$ in the jth hour $(j = 1,\ldots,24)$ of the ith day $(i = 1,\ldots,D)$ for the dth turbine slot $(d = 1,\ldots,3)$ of the cth unit $(c = 1,\ldots,3)$.

Then an estimate of total passage in units 1-3 can be expressed as

$$\hat{T}_{1-3} = \sum_{c=1}^{3}\sum_{d=1}^{3}\sum_{i=1}^{D}\sum_{j=1}^{24}\frac{K}{k}\sum_{l=1}^{k} x_{cdijl} \tag{4}$$

with associated variance estimator

$$Var\left(\hat{T}_{1-3}\middle|T_{1-3}\right) = \sum_{c=1}^{3}\sum_{d=1}^{3}\sum_{i=1}^{D}\sum_{j=1}^{24}\left[\frac{K^2\left(1-\frac{k}{K}\right)s^2_{x_{cdij}}}{k}\right] \tag{5}$$

and where

$$s^2_{x_{cdij}} = \frac{\sum_{i=1}^{k}\left(x_{cdijl} - \bar{x}_{cdij}\right)^2}{(k-1)},$$

$$\bar{x}_{cdij} = \frac{\sum_{l=1}^{k} x_{cdijl}}{k}.$$

For turbine units 4-6, total smolt passage would be estimated by

$$\hat{T}_{4-6} = \sum_{h=4}^{6}\sum_{i=1}^{D}\sum_{j=1}^{24}\frac{K}{k}\sum_{l=1}^{k} x_{hijl} \tag{6}$$

where x_{hijl} was defined earlier.

A negatively biased variance estimator for $\hat{T}_{4-6}$ can be calculated as

$$V\tilde{a}r\left(\hat{T}_{4-6}\middle|T_{4-6}\right)=\sum_{h=4}^{6}\sum_{i=1}^{D}\sum_{j=1}^{24}\frac{K^2\left(1-\dfrac{k}{K}\right)s_{hij}^2}{k}. \tag{7}$$

Variance estimator (7) considers only the within-hour sampling error and ignores the slot-to-slot variance component.

A positively biased variance estimator can be constructed by considering the 9 intake slots at unit 4-6 as having been randomly sampled at 3 of the 9 locations. In this case, the variance for $\hat{T}_{4-6}$ can be estimated by the expression

$$V\tilde{a}r\left(\hat{T}_{4-6}\middle|T_{4-6}\right)=\frac{9^2\left(1-\dfrac{3}{9}\right)s_{\hat{T}_d}^2}{3}+\frac{9\sum_{d=1}^{3}V\hat{a}r\left(\hat{T}_d\middle|T_d\right)}{3} \tag{8}$$

where

$$s_{\hat{T}_d}^2=\frac{\sum_{d=1}^{3}\left(\hat{T}_d-\hat{\bar{T}}\right)^2}{(3-1)},$$

$$\hat{\bar{T}}=\frac{\sum_{d=1}^{3}\hat{T}_d}{d},$$

$$\hat{T}_d=\sum_{i=1}^{D}\sum_{j=1}^{24}\frac{K}{k}\sum_{l=1}^{k}x_{dijl},$$

for

x_{dijl} = weighted number of fish detected in the lth sampling interval $(l=1,\ldots,k)$ of the jth hour $(j=1,\ldots,24)$ of the ith day $(i=1,\ldots,D)$ at the dth slot $(d=1,\ldots,3)$

and where

$$\hat{Var}\left(\hat{T}_d \middle| T_d\right) = \sum_{i=1}^{D}\sum_{j=1}^{24} \frac{K^2\left(1-\frac{k}{K}\right)s_{x_{dij}}^2}{k},$$

$$s_{x_{dij}}^2 = \frac{\sum_{l=1}^{k}\left(x_{dijl} - \bar{x}_{dij}\right)^2}{(k-1)},$$

$$\bar{x}_{dij} = \frac{\sum_{l=1}^{k} x_{dijl}}{k}.$$

Variance estimator (8) will tend to overestimate the sampling variance for it includes unit-to-unit variance which was eliminated by stratified sampling of each of the turbine units.

Total powerhouse passage is then estimated by

$$\hat{T} = \hat{T}_{1-3} + \hat{T}_{4-6}$$

with associated variance estimator

$$\hat{Var}\left(\hat{T}_{1-3} \middle| T_{1-3}\right) + \hat{Var}\left(\hat{T}_{4-6} \middle| T_{4-6}\right)$$

or

$$\hat{Var}\left(\hat{T}_{1-3} \middle| T_{1-3}\right) + \tilde{Var}\left(\hat{T}_{4-6} \middle| T_{4-6}\right).$$

Subsequent calculations of smolt passage performance based on $\hat{T}$ should consider both liberal and conservative estimators in the analyses.

C. Estimating Smolt Passage Into the SBC

In 1999, the surface collector will have two entrances, each will receive 2 squinted-pair of single-beam hydroacoustic transducers to estimate fish passage. Each squinted-pair will be used to sample a spatial stratum formed by vertically bisecting the SBC openings (Figure 2). Thus, each squinted-pair of transducers will sample one-half of the vertical openings in the SBC. This sampling will be assumed to be stratified simple random sampling within hours, within each of the vertically bisectional areas of an SBC opening. The estimate of total SBC passage can then be expressed as

$$\hat{C} = \sum_{i=1}^{2}\sum_{j=1}^{2}\sum_{k=1}^{D}\sum_{l=1}^{24}\frac{N_c}{n_c}\sum_{g=1}^{n_c} y_{ijklg}$$

where

y_{ijklg} = weighted number of smolt in the *g*th sampling interval $(g = 1, \ldots, N_c)$ of the *l*th hour $(l = 1, \ldots, 24)$ of the *k*th day $(k = 1, \ldots, D)$ of the *j*th bisectional area $(j = 1,2)$ of the *i*th entrance $(i = 1,2)$ of the SBC;

n_c = number of sampling intervals insonified during an hour at a collector entrance;

N_c = number of possible sampling intervals within an hour at a collector entrance.

Nominally, n_c = 3 2.5-minute intervals out of a possible N_c = 24 per hour.

The estimated variance of $\hat{C}$ is expressed as

$$V\hat{a}r\left(\hat{C}\middle|C\right) = \sum_{i=1}^{2}\sum_{j=1}^{2}\sum_{k=1}^{D}\sum_{l=1}^{24}\left[\frac{N_c^2\left(1-\frac{n_c}{N_c}\right)s_{y_{ijkl}}^2}{n_c}\right]$$

where

$$s_{y_{ijkl}}^2 = \frac{\sum_{g=1}^{n_c}\left(y_{ijklg} - \bar{y}_{ijkl}\right)^2}{\left(n_c - 1\right)}$$

and where

$$\bar{y}_{ijkl} = \frac{\sum_{g=1}^{n_c} y_{ijklg}}{n_c}.$$

The y_{ijklg} are the weighted fish counts after expanding to the bisectional area of an SBC entrance. Furthermore, the y_{ijklg} are the numbers of smolt designated as moving into the SBC entrance based on hydroacoustic scans of smolt movement patterns. Figure 4 depicts a schematic of the squinted-pair transducer configuration and possible fish traces.

Define

z_1 = number of fish with trace 1,
z_2 = number of fish with trace 2,
z_3 = number of fish with trace 3,
z_4 = number of fish with trace 4,

and

$$Z = z_1 + z_2 + z_3 + z_4 \,.$$

The number of fish entering the SBC will be estimated by the sum $z_1 + z_3$, as was used in 1996 –1998. Radiotelemetry information from 1996 suggests only 1-2% of the fish entering the SBC may be exiting. Hence, the y_{ijklg} are based on values of $z_1 + z_3$ in the sampling intervals measured.

IV. Performance Measures

Nine different performance measures will be estimated from the hydroacoustic data collected in 1999 at Lower Granite Dam. Below each response and its associated variance are described. These performance measures will be estimated season-wide, for day and night periods, and on a weekly basis.

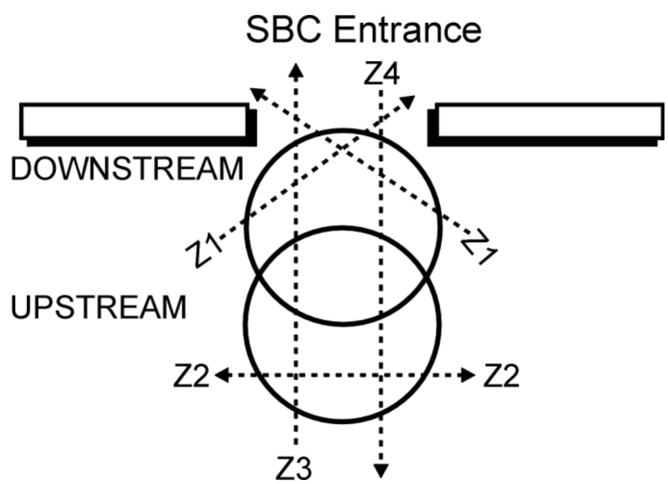
A. Spill Performance

1. *Spill Efficiency*

Spill efficiency (SE) at Lower Granite Dam will be defined as the proportion

$$SE = \frac{S}{S + T + C}$$

Figure 4. Schematic of squinted-pair of single-beam transducers and the four possible trace types (called Z-types).



where

S = total smolt passage in spillway;

T = total turbine passage (i.e., guided and unguided);

C = total SBC passage.

An estimate of SE can then be calculated as

$$\hat{S}E = \frac{\hat{S}}{\hat{S} + \hat{T} + \hat{C}}.$$

Using the Delta method (Seber 1982: 7-11), a variance estimator can be approximated by

$$V\hat{a}r\left(\hat{S}E \middle| SE\right) = \hat{S}E^2\left(1 - \hat{S}E\right)^2\left[\frac{V\hat{a}r\left(\hat{S}\right)}{\hat{S}^2} + \frac{V\hat{a}r\left(\hat{T}\right) + V\hat{a}r\left(\hat{C}\right)}{\left(\hat{T} + \hat{C}\right)^2}\right].$$

2. ***Spill Effectiveness***

Spill effectiveness (SEF) will be defined as

$$SEF = \frac{\left(\frac{S}{S + T + C}\right)}{\left(\frac{f}{F}\right)} = SE \cdot \left(\frac{F}{f}\right)$$

where

F = total water volume discharged at the dam,

f = total water volume discharged through the spillway.

As estimator of spill effectiveness can then be written as

$$S\hat{E}F = \hat{S}E \cdot \left(\frac{F}{f}\right)$$

with a variance estimator of

$$Var\left(S\hat{E}F \middle| SEF\right) = V\hat{a}r\left(\hat{S}E \middle| SE\right)\left(\frac{F}{f}\right)^2 .$$

3. ***Spillbay Effectiveness***

The relative performance of each of the spillbays can be calculated to determine the most appropriate spillbay to effectively move smolt through the dam. Define spillbay effectiveness for the *i*th spillbay as

$$SBEF_i = \frac{\left(\frac{S_i}{f_i}\right)}{\left[\frac{\left(\sum_{i=1}^{7} S_i\right)}{\left(\sum_{i=1}^{7} f_i\right)}\right]}$$

where f_i = flow volume through the *i*th spillbay. Then an estimator of spillbay effectiveness can be expressed as

$$\begin{aligned} S\hat{B}EF_i &= \frac{\hat{S}_i}{\left(\sum_{i=1}^{7} \hat{S}_i\right)} \cdot \frac{\left(\sum_{i=1}^{7} f_i\right)}{f_i} \\ &= \hat{R}S_i \left(\frac{\sum_{i=1}^{7} f_i}{f_i}\right) \end{aligned}$$

where $\hat{R}S_i$ is the relative spillbay passage for the *i*th spillbay $(i = 1, \ldots, 7)$. An associated variance estimator for spillbay effectiveness can then be written as

$$V\hat{a}r\left(S\hat{B}EF \middle| SBEF\right) = \hat{R}S_i\left(1 - \hat{R}S_i\right)^2 \left[\frac{V\hat{a}r\left(\hat{S}_i\right)}{\hat{S}_i^2} + \frac{\sum_{j \neq i} Var\left(S_j\right)}{\left(\sum_{j \neq i} \hat{S}_i\right)^2}\right] \cdot \left(\frac{\sum_{i=1}^{7} f_i}{f_i}\right)^2 .$$

Spillbay effectiveness will be calculated for each of the seven $(i = 1, \ldots, 7)$ operating spillbays at Lower Granite Dam in 1999.

B. SBC Performance

1. *SBC Efficiency*

The SBC efficiency (R) expresses the fraction of the fish entering the SBC relative to the total number of fish passing into either the collector or other designated routes. Three different measures of SBC efficiency relative to powerhouse passage will be calculated in 1999. These are

$$\hat{R}_{4-5} = \frac{\hat{C}}{\hat{C} + \hat{T}_{4-5}},$$
$$\hat{R}_{4-6} = \frac{\hat{C}}{\hat{C} + \hat{T}_{4-6}},$$
$$\hat{R}_{1-6} = \frac{\hat{C}}{\hat{C} + \hat{T}_{1-6}},$$

where $\hat{T}$ is defined by smolt passage at different turbine units. The variance of $\hat{R}_i$ can be approximated by the expression

$$V\hat{a}r\left(\hat{R}_i \middle| R_i\right) = \hat{R}_i^2\left(1 - \hat{R}_i\right)^2 \left[\frac{V\hat{a}r\left(\hat{C}\right)}{\hat{C}^2} + \frac{V\hat{a}r\left(\hat{T}_i\right)}{\hat{T}_i^2}\right]$$

for any turbine designation *i*.

2. *SBC Overall Efficiency*

The fractional passage of smolt through the SBC relative to the total project $\left(R_{ALL}\right)$ is defined as

$$R_{ALL} = \frac{C}{C+S+T}$$

and can be estimated by

$$\hat{R}_{ALL} = \frac{\hat{C}}{\hat{C}+\hat{S}+\hat{T}}.$$

Using the Delta method, the variance for $\hat{R}_{ALL}$ can be approximated by

$$V\hat{a}r\left(\hat{R}_{ALL} \middle| R_{ALL}\right) = R_{ALL}^2\left(1-\hat{R}_{ALL}\right)^2\left[\frac{V\hat{a}r\left(\hat{C}\right)}{\hat{C}^2} + \frac{V\hat{a}r\left(\hat{S}\right)+V\hat{a}r\left(\hat{T}\right)}{\left(\hat{S}+\hat{T}\right)^2}\right].$$

3. ***SBC Effectiveness***

Effectiveness of the SBC with regard to fish passed per unit of water will be defined according to the formula

$$E_{1-6} = \frac{\dfrac{C}{f}}{\dfrac{\left(C+T_{1-6}\right)}{F}}$$

and estimated by the formula

$$\hat{E}_{1-6} = \frac{\left(\dfrac{\hat{C}}{\hat{C}+\hat{T}_{1-6}}\right)}{\left(\dfrac{f}{F}\right)} = \hat{R}_{1-6}\cdot\frac{F}{f}$$

where

F = total water volume discharged through the SBC and turbine units 1-6;

f = total water volume discharged through the SBC.

The variance of $\hat{E}_{1-6}$ can then be calculated as

$$\hat{Var}\left(\hat{E}_{1-6}\middle|E_{1-6}\right) = \hat{Var}\left(\hat{R}_{1-6}\right)\cdot\left(\frac{F}{f}\right)^2 .$$

4. ***SBC Entrance Efficiency***

The entrance efficiency of the SBC will be calculated based on the fish traces from the squinted-pairs of single-beam transducers at the SBC entrance. The overall entrance efficiency is defined as

$$N = \frac{Z_{1,3}}{Z_{1,3} + Z_{2,4}}$$

where the $Z_{1,3}$ is defined by traces $Z_1 + Z_3$ and $Z_{2,4}$ is defined as traces $Z_2 + Z_4$. The $Z_{1,3}$ has also been expressed as C and estimated by $\hat{C}$. The fish near the SBC entrance that do not enter will be estimated by the quantity

$$\hat{U} = \sum_{i=1}^{2}\sum_{j=1}^{2}\sum_{k=1}^{D}\sum_{l=1}^{24}\frac{N_c}{n_c}\sum_{g=1}^{n_c}u_{ijk\,lg}$$

where $u_{ijk\,lg}$ = weighted number of smolt in the gth sampling interval $(g = 1,\ldots,N_c)$ of the lth hour $(l = 1,\ldots,24)$ of the kth day $(k = 1,\ldots,D)$ of the jth bisectional area $(j = 1,2)$ of the ith entrance $(i = 1,2)$ that do not enter the SBC.

The u's are calculated as the sum of traces $Z_2 + Z_4 = Z_{2,4}$ for a sampling interval. The estimated variance of $\hat{U}$ is expressed as

$$\hat{Var}\left(\hat{U}\middle|U\right) = \sum_{i=1}^{2}\sum_{j=1}^{2}\sum_{k=1}^{D}\sum_{l=1}^{24}\left[\frac{N_c^2\left(1-\frac{n_c}{N_c}\right)s_{u_{ijkl}}^2}{n_c}\right]$$

where

$$s_{u_{ijkl}}^2 = \frac{\sum_{g=1}^{n_c}\left(u_{ijklg} - \bar{u}_{ijkl}\right)}{\left(n_c - 1\right)}$$

and where

$$\bar{u}_{ijkl} = \frac{\sum_{g=1}^{n_c} u_{ijklg}}{n_c}.$$

An estimate of SBC entrance efficiency can then be calculated as

$$\hat{N} = \frac{\hat{C}}{\hat{C} + \hat{U}}.$$

Using the Delta method, an approximate variance estimator for $\hat{N}$ can be calculated by

$$V\hat{a}r\left(\hat{N}\middle|N\right) = \hat{N}^2\left(1 - \hat{N}\right)^2\left[\frac{V\hat{a}r\left(\hat{C}\right)}{\hat{C}^2} + \frac{V\hat{a}r\left(\hat{U}\right)}{\hat{U}^2} - \frac{2C\hat{o}v\left(\hat{C},\hat{U}\right)}{\hat{C}\hat{U}}\right]$$

where

$$C\hat{o}v\left(\hat{C},\hat{U}\right) = \sum_{i=1}^{2}\sum_{j=1}^{2}\sum_{k=1}^{D}\sum_{l=1}^{24}\left[\frac{N_c^2\left(1 - \frac{n_c}{N_c}\right)s_{y_{ijkl}u_{ijkl}}}{n_c}\right]$$

and where

$$s_{y_{ijkl}u_{ijkl}} = \frac{\sum_{g=1}^{n_c}\left(y_{ijklg} - \bar{y}_{ijkl}\right)\left(u_{ijklg} - \bar{u}_{ijkl}\right)^2}{\left(n_c - 1\right)}.$$

Entrance efficiency will be computed for the entire SBC (i.e., *N*) as well as for each of the two SBC entrances (i.e., $\hat{N}_i, i = 1,2$).

V. Literature Cited

Seber, G. A. F. 1982. The estimation of animal abundance. Macmillan. New York, NY. 654 pp.